

Math 2016.05

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Exercise 1. Mr Lee had some red and white paint. $\frac{1}{3}$ of the paint was red. He used $\frac{1}{2}$ of the red paint and $\frac{5}{6}$ of the white paint. In the end, he had a total of $10l$ of paint left. How much paint did he have at first?

Answer 1. Let's set up a table to show the Before and After cases.

Time	Red	White	Comments
Before	$\frac{1}{3}$	$\frac{2}{3}$	Before
Action	$\frac{1}{2}$	$\frac{5}{6}$	Uses this portion of each paint
Remains	$\frac{1}{2}$	$\frac{1}{6}$	This portion of each paint remains
After	$\frac{1}{3} \left(\frac{1}{2}\right)$	$\frac{2}{3} \left(\frac{1}{6}\right)$	Amount of each paint remaining
After	$\frac{1}{6}$	$\frac{2}{18} = \frac{1}{9}$	Amount of each paint remaining

We know that $10l$ of paint remains. We also know the proportions of each paint remaining from the "After" row of the table above. We can now solve for original amount of paint by working backwards from what we know at the "After" stage.

The idea is to use this proportion-
(Amount Used) x (Original Amount) = Remaining Amount

Let's use x to mean the Original Amount. Then using the amounts found above- we can express this equation as follows...

$$\left(\frac{1}{6} + \frac{1}{9}\right) * x = 10l$$

$$\left(\frac{1}{6} * \frac{9}{9} + \frac{1}{9} * \frac{6}{6}\right) * x = 10$$

$$\left(\frac{9}{54} + \frac{6}{54}\right) * x = 10$$

$$\left(\frac{15}{54}\right) * x = 10$$

$$x = \frac{540}{15} = 36$$

Mr Lee had $36l$ of paint at first.

Doublecheck 1. Let's check if $36l$ of paint is correct.

If $1/3$ of the paint was red and $2/3$ of the paint was white- that means $12l$ was red and $24l$ was white. Let's use $1/2$ of the red paint and $5/6$ of the white paint. That uses up $12*(1/2) = 6l$ of the red paint and $24*(5/6)=20l$ of the white paint. This leaves $12-6= 6l$ of red paint and $24-20 = 4l$ of white paint remaining. $6 + 4 = 10l$. This checks the answer.